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The Λ_b LIFETIME IN THE LIGHT FRONT QUARK MODEL.

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Abstract

The enhancement of the Λ_b decay width relative to B decay one due to the difference of Fermi motion effects in Λ_b and B is calculated in the light–front quark model with the simplifying assumption that Λ_b consists of the heavy quark and light scalar diquark. In order to explain the large deviation from unity in the experimental result for $\tau(\Lambda_b)/\tau(B)$, it is necessary that diquark be light and the ratio of the squares of the Λ_b and B wave functions at the origin be ≤ 1.

The lifetimes of the b flavoured hadrons H_b are related both to the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$ and to dynamics of H_b decays. In the limit $m_b \to \infty$ the light quarks do not affect the decay of the heavy quark, and thus the lifetimes of all b hadrons must be equal. The account of the soft degrees of freedom generates the preasymptotic corrections which, however, have non-significant impact on the lifetimes and various branching fractions of B and B_s mesons. Inclusive H_b decays can be treated with the help of an operator product expansion (OPE) combined with the heavy quark expansion [1]. The OPE approach predicts that all corrections to the leading QCD improved parton terms appear at the order $1/m_b^2$ and beyond. Thus mesons and baryons containing b quark are expected to have lifetimes differing by no more than a few percent. The result of this approach for the Λ_b lifetime is puzzling because it predicts that $(\tau(\Lambda_b)/\tau(B))_{OPE} = 0.98 + \mathcal{O}(1/m_b^3)$ [2], whereas the experimental findings suggest a very much reduced fraction $(\tau(\Lambda_b)/\tau(B))_{exp} = 0.78 \pm 0.04$ [3] or conversely a very much enhanced decay rate. The decay rates of B and Λ_b are $\Gamma(B) = 0.63 \pm 0.02$ ps^{-1} and $\Gamma(\Lambda_b) = 0.83 \pm\ 0.05$ ps^{-1} differing by $\Delta\Gamma(\Lambda_b) = 0.20 \pm\ 0.05$ ps^{-1}. The four–fermion processes of weak scattering and Pauli interference could explain, under certain conditions, only $(13 \pm 7)\%$ of this difference [4] (see, however, [2], [5]). In spite of great efforts of experimental activity the Λ_b lifetime

remains significantly low which continues to spur theoretical activity. In this respect, the use of phenomenological models, like the constituent quark model, could be of interest as a complementary approach to the OPE resummation method.

In this paper we shall compute the preasymptotic effects for the Λ_b lifetime in the framework of the light–front (LF) quark model, which is a relativistic constituent quark model based on the LF formalism. In Ref. [6] this formalism has been used to establish a simple quantum mechanical relation between the inclusive semileptonic decay rate of the B meson and that of a free b quark. The approach of [6] relies on the idea of duality in summing over the final hadronic states. It has been assumed that the sum over all possible charm final states X_c can be modelled by the decay width of an on–shell b quark into on–shell c quark folded with the b–quark distribution function $f^b_B(x,p^2_\perp) = |\varphi^b_B(x,p^2_\perp)|^2$ The latter represents the probability to find b quark carrying a LF fraction x of the hadron momentum and a transverse relative momentum squared $p^2_\perp$. For the semileptonic rates the abovementioned relation takes the form

$$\frac{d\Gamma_{SL}(B)}{dt} = \frac{d\Gamma^b_{SL}}{dt} R_B(t), \tag{1}$$

where $d\Gamma^b_{SL}/dt$ is the free quark differential decay rate, $t = q^2/m_b^2$, q being the 4–momentum of the W boson, and $R_B(t)$ incorporates the nonperturbative effects related to the Fermi motion of the heavy quark inside the hadron. The expression for $d\Gamma^b_{SL}/dt$ for the case of non–vanishing lepton masses is given *e.g.* in [6]. $R(t)$ in (1) is obtained by integrating the bound–state factor $\omega(t,s)$ over the allowed region of the invariant hadronic mass M_{X_c}:

$$R_B(t) = \int\limits_{s_{min}}^{s_{max}} ds \omega(t,s), \tag{2}$$

where $s = M^2_{X_c}/m_b^2$ and

$$\omega(t,s) = m_b^2 x_0 \; \frac{\pi m_b}{q^+} \; \frac{|\mathbf{q}|}{|\tilde{\mathbf{q}}|} \int\limits_{x_1}^{min[1,x_2]} dx \; |\varphi^b_B(x,p^{*2}_\perp)|^2. \tag{3}$$

In Eq. (3) $x_0 = m_b/M_B$, $p^{*2}_\perp = m_b^2(\xi(1-\rho-t) - \xi^2 t - 1)$ with $\xi = \frac{xM_B}{q^+}$, and $\rho = (m_c/m_b)^2$, and the limits of integration $x_{1,2}$ are given by $x_{1,2} = x_0 q^+/\tilde{q}^\pm$. The plus component $q^+ = q_0 + |\mathbf{q}|$ is defined in the B meson rest frame whereas $\tilde{q}^\pm = \tilde{q}_0 \pm |\tilde{\mathbf{q}}|$ are defined in the b quark rest frame. In Eq. (2) the region of integration over s is defined through the condition $x_1 \le min[1,x_2]$, *i.e.* $s_{max} = x_0^{-2}(1 - x_0\sqrt{t})^2$. For other details see [6].

In the quark model the Fermi motion effect is due to the interaction with valence quark. The LF wave function $\varphi^b_B(x,p^2_\perp)$ is defined in terms of the equal time radial wave function $\psi_B(p^2)$ as [7] $\varphi^b_B(x,p^2_\perp) = \frac{\partial p_z}{\partial x}\frac{\psi_B(p^2)}{\sqrt{4\pi}}$, where $p^2 = p^2_\perp + p^2_z$, $p_z = \left(x - \frac{1}{2}\right) M_0 + \frac{m^2_{sp} - m_b^2}{2M_0}$, $M_0 = \sqrt{p^2 + m_b^2} + \sqrt{p^2 + m^2_{sp}}$, and m_{sp} is the constituent mass of the spectator quark. Explicit expression for $\frac{\partial p_z}{\partial x}$ can be found in [8]. In what follows the B meson orbital wave function is assumed to be the Gaussian function as

$$\psi_B(p^2) = \left(\frac{1}{\beta_{b\bar{d}}\sqrt{\pi}}\right)^{\frac{3}{2}} \exp\left(-\frac{p^2}{2\beta^2_{b\bar{d}}}\right), \tag{4}$$

where the parameter $1/\beta_{b\bar{d}}$ defines the confinement scale. We take $\beta_{b\bar{d}} = 0.45$ GeV that is very close to the variational parameter 0.43 GeV found in the Isgur–Scora model [9] and corresponds to value of the QCD parameter $-\lambda_1 =< p_b^2 >= 0.2$ GeV^2. For $|\Psi_B(0)|^2$, the square of the wave function at the origin, we have $|\Psi_B(0)|^2 = 1.64 \cdot 10^{-2}$ GeV^3. This value compares favourably with the estimation in the constituent quark ansätz [10] $|\Psi_B(0)|^2 = M_B f_B^2/12 = (1.6 \pm 0.7) \cdot 10^{-2}$ GeV^3 for $f_B = 190 \pm 40$ MeV.

The same formulae can be also applied for nonleptonic B decay widths (corresponding to the underlying quark decays $b \to cq_1q_2$) thus making it possible to calculate the B lifetime [11]. The lepton pair is substituted by a quark pair, so that $d\Gamma^b_{SL}/dq^2$ is replaced by $d\Gamma^b_{NL}/dq^2 = \eta |V_{q_1q_2}|^2 d\Gamma^b_{SL}/dq^2$, where (in the limit $N_c \to \infty$) $\eta = \frac{3}{2}(c_+^2 + c_-^2)$, with c_- and $c_+ = c_-^{-1/2}$ being the standard short distance QCD enhancement and suppression factors in a color antitriplet and sextet, respectively. We take $c_+ = 0.84$, $c_- = 1.42$.

The constituent quark masses are the free parameters in our model. We have found that the τ_{Λ_b}/τ_B ratio is rather stable with respect to the precise values of the heavy quark masses m_b and m_c provided $m_b - m_c \geq 3.5$ GeV. From now on we shall use the reference values $m_b = 5.1$ GeV and $m_c = 1.5$ GeV. The value of the CKM parameter $|V_{cb}|$ cancels in the ratio τ_{Λ_b}/τ_B, but is important for the absolute rates. Details of our calculations of $\Gamma(B)$ are given in Table 1 for the three different values of the constituent mass m_{sp}. The values $m_{sp} \sim 300$ (200) MeV are usually used in non-relativistic (relativized) quark models [9],[12]. We have also considered a very low constituent quark mass $m_{sp} = 100$ MeV to see how much we can push up the theoretical prediction of the $\Gamma(\Lambda_b)/\Gamma(B)$, see below. All the semileptonic widths include the pQCD correction as an overall reduction factor equal to 0.9. Following Ref. [8] we have included the transitions to baryon-antibaryon ($\Lambda_c\bar{N}$ and $\Xi_{cs}\bar{\Lambda}$) pairs. In addition we have added BR $\approx 1.5\%$ for the Cabbibo–suppressed $b \to u$ decays with $|V_{ub}/V_{cb}| \sim 0.1$. The value of $|V_{cb}|$ is defined by the condition that the calculated B lifetime is 1.56 ps.

Now we turn to the calculation of the Λ_b decay rate. We shall analyze the inclusive semileptonic and non–leptonic Λ_b rates on the simplifying asumption that Λ_b is composed of a heavy quark and a light scalar diquark with the effective mass m_{ud}. Then the treatment of the inclusive Λ_b decays is simlified to a great extent and we can apply the model considered above with the minor modifications. For the heavy–light diquark wave function ψ_{Λ_b} we again assume the Gaussian ansätz with the oscillator parameter β_{bu}. The width of Λ_b can be obtained from that of B by the replacements $M_B \to M_{\Lambda_b}$, $m_{sp} \to m_{ud}$ and $\beta_{b\bar{d}} \to \beta_{bu}$. Note that the latter two replacements change $f^b_{\Lambda_b}$, the b quark distribution function inside the Λ_b, in comparison with f^b_B.

The inclusive nonleptonic channels for Λ_b are the same as for B meson except for the decays into baryon–antibaryon pairs which are missing in case of Λ_b. The absence of this decay channel leads to the reduction of $\Gamma(\Lambda_b)$ by $\approx 7\%$. This reduction can not be compensated by the phase space enhancement in Λ_b. The only way to get an enchancement of Γ_{Λ_b} is to enhance its non–leptonic rates. Altarelly *et al.* [13] have suggested the increase of the non–leptonic rates could be due to the phenomenological factor $(M_{H_b}/m_b)^5$, then the 6% difference between M_{Λ_b} and M_B is enough to explain the experimentally observed enhancement. In our approach, the only distinction between the two lifetimes, τ_{Λ_b} and τ_B, can occur due to the difference of Fermi motion effects encoded in $f^b_{\Lambda_b}$ and f^b_B.

The $f^b_{\Lambda_b}$ is defined by the two parameters, m_{ud} and β_{ub}. The later quantity can be translated into the ratio of the squares of the wave functions determining the probability to find a light quark at

the location of the b quark inside the Λ_b baryon and B meson, *i.e.*

$$r = \frac{|\Psi_{\Lambda_b}(0)|^2}{|\Psi_B(0)|^2} = \left(\frac{\beta_{bu}}{\beta_{b\bar{d}}}\right)^3 . \tag{5}$$

Estimates of the parameter r using the non–relativistic quark model or the bag model [14], [15], [16] or QCD sum rules [17] are typically in the range $0.1 - 0.5$. On the other hand, Rosner has estimated the heavy–light diquark density at zero separation in Λ_b from the ratio of hyperfine splittings between Σ_b and Σ_b^* baryons and B and B^* mesons and finds [4]

$$r = \frac{4}{3} \cdot \frac{m^2_{\Sigma_b^*} - m^2_{\Sigma_b^*}}{m^2_{B^*} - m^2_B}. \tag{6}$$

This lead to $r \sim 0.9 \pm 0.1$, if the baryon splitting is taken to be $m^2_{\Sigma_b^*} - m^2_{\Sigma_b} \sim m^2_{\Sigma_c^*} - m^2_{\Sigma_c} = (0.384 \pm 0.035)$ GeV^2, or even to $r \sim 1.8 \pm 0.5$, if the surprisingly small and not confirmed yet DELPHI result $m_{\Sigma_b^*} - m_{\Sigma_b} = (56 \pm 16)$ MeV [18] is used.

On the other hand, the width $\Gamma(\Lambda_b)$ and hence the ratio $\tau(\Lambda_b)/\tau(B)$ is very sensitive to the choice of m_{ud} and r. In order to study the dependence on m_{ud} and r we keep the values of the quark masses m_b and m_c fixed and vary the wave function ratio in the range $0.3 \leq r \leq 2.3$ that corresponds to 0.3 GeV $\leq \beta_{bu} \leq$ 0.6 GeV. We take two representative values for the diquark mass: $m_{ud} = m_u + m_d$ corresponding to zero binding approximation and $m_{ud} = m_* \approx \frac{1}{2}(m_u + m_d - m_\pi)$. In the latter relation inspired by the quark model, the factor 1/2 arises from the diferent color factors for u and $\bar{d}$ in the π–meson (a triplet and antitriplet making a singlet) and u and d in the the Λ_b (two triplets making an antitriplet).

In Fig. 1 we compare one–dimensional distribution functions

$$F^b_{\Lambda_b}(x) = \pi \int_0^\infty dp_\perp^2 f^b_{\Lambda_b}(x, p_\perp^2) \tag{7}$$

with that of the B meson. These functions exhibit a pronounced maximum at $m_b/(m_b + m_{sp})$ (in case of the B meson) and $m_b/(m_b + m_{ud})$ (in case of Λ_b). The width of F_{Λ_b} depends on β_{bu} and goes to zero when $\beta_{bu} \to 0$. Note that the calculated branching fractions of Λ_b show marginal dependence on the choice of the model parameters; they are ~ 11.5 % for the semileptonic $b \to ce\nu_e$ transitions, $\sim 2.8\%$ for $b \to c\tau\nu_\tau$, $\sim 50\%$ for the nonleptonic $b \to cd\bar{u}$ transitions, and $\sim 16\%$ for $b \to c\bar{c}s$ transitions.

Our results for τ_{Λ_b}/τ_B are shown in Table 2 and Fig. 2. We have also included the contribution of four-quark operators calculated using the factorization approach and the description of the baryon relying on quantum mechanics of only the constituent quarks. This contribution leads to a small enhancement of the Λ_b decay rate by an amount

$$\Delta\Gamma^{4q}(\Lambda_b) = \frac{G_F^2}{2\pi} |\Psi_{bu}(0)|^2 |V_{ud}|^2 |V_{cb}|^2 m_b^2 (1-\rho)^2 [c_-^2 - (1+\rho)c_+(c_- - c_+/2)]. \tag{8}$$

This contribution scales like β^3_{bu} and varies between 0.01 ps^{-1} and 0.03 ps^{-1} when β_{bu} varies between 0.35 and 0.55 GeV.

The quantity τ_{Λ_b}/τ_B is particular sensitive to the light quark mass m_{sp}. We observe that to decrease the theoretical prediction for τ_{Λ_b} requires to decrease the value of the hadronic parameter

r in (5) to 0.3-0.5 and the value of m_{sp} to $\sim$ 100 MeV. For example, assuming that $r \sim 0.3$ we find that the lifetime ratio is decreased from 0.88 to 0.81 if m_{sp} is reduced from 300 MeV to 100 MeV and the diquark mass is chosen as $m_{ud} = m_u + m_d$. For the diquark mass $m_{ud} \sim m_*$ the ratio is almost stable (~ 0.8), so that reducing of the diquark mass produces a decrease of the lifetime ratio by 1%, 5%, and 8% for $m_{sp} = 100,\ 200,$ and 300 MeV, respectively. Varying the spectator quark mass in a similar way we find that for the "central value" $r \sim 1$ the lifetime ratios are reduced from 0.93 to 0.88 for $m_{ud} = m_u + m_d$ and remain almost stable (~ 0.86) for $m_{ud} \sim m_*$. For the largest possible value of r suggested in [4], $r \sim 2.3$, the lifetime ratios are reduced from 0.97 to 0.94 in the former case and remain almost stable $\sim 0.91 - 0.93$ in the latter case.

If the current value of $(\tau(\Lambda_b)/\tau(B))_{exp}$ persists, the most likely its explanation is that some hadronic matrix elements of four–quark operators are larger than the naive expectation (8) [2]. A recent lattice study of Ref. [5] suggests that the effects of weak scattering and interference can be pushed at the $\approx 8\%$ level for $r = 1.2 \pm 0.2$ *i.e.* for the value of r that is significantly larger that most quark model predictions but smaller than the upper Rosner estimation. If a significant fraction $\sim 50\%$ of the discrepancy between the theoretical prediction for τ_{Λ_b}/τ_B and the experimental result can be accounted for the spectator effects then the reminder of the discrepancy can be explained by the preasymptotic effect due to Fermi motion of the b quark inside Λ_b. Indeed, choosing the quite reasonable values of the spectator quark mass $m_{sp} = 200$ MeV and the diquark mass $m_{ud} = 250$ MeV and subtracting the small contribution (8) we find that the Fermi motion effect produces for $r = 1.2 \pm 0.2$ an additional reduction of τ_{Λ_b} by $12 \pm 2\%$.

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Table 1. The branching fractions (in per cent) for the inclusive semileptonic and nonleptonic B decays calculated within the LF quark model for the several values of the constituent quark mass m_{sp}. The heavy quark masses are $m_b = 5.1$ GeV, $m_c = 1.5$ GeV. The oscillator parameter in Eq. (4) is $\beta_{b\bar{d}} = 0.45$ GeV. The values of $|V_{cb}|$ in units of $10^{-3}\sqrt{1.56 \text{ ps}/\tau^{(exp)}(B)}$ are also reported.

	$m_{sp} = 100$ MeV	$m_{sp} = 200$ MeV	$m_{sp} = 300$ MeV
$b \to ce\nu_e$	10.65	10.98	11.46
$b \to c\mu\nu_\mu$	10.59	10.93	11.40
$b \to c\tau\nu_\tau$	2.47	2.51	2.57
$b \to cd\bar{u}$	47.88	47.88	47.52
$b \to c\bar{c}s$	14.07	14.31	14.63
$b \to cs\bar{u}$	2.94	3.09	3.32
$B \to \Xi_{cs}\bar{\Lambda}_c$	2.22	1.83	1.43
$B \to \Lambda_c\bar{N}$	7.70	6.91	6.02
$b \to u$	1.47	1.54	1.66
$\|V_{bc}\|$	38.3	39.3	40.7

Table 2. The LF quark model results for the ratio τ_{Λ_b}/τ_B calculated for different sets of paremeters. The diquark masses are given in units of MeV, whereas β_{bu} are in units of GeV.

	$m_{sp} = 100$ MeV		$m_{sp} = 200$ MeV		$m_{sp} = 300$ MeV	
$\beta_{bu} \setminus m_{ud}$	200	150	400	250	600	400
0.3	0.807	0.795	0.843	0.792	0.885	0.803
0.45	0.877	0.867	0.901	0.857	0.932	0.859
0.6	0.937	0.929	0.951	0.913	0.970	0.906

Figure captions

Figure 1. The distribution functions $F^b_B(x)$ (solid line) and $F^b_{\Lambda_b}(x)$ (thin lines) defined by Eq. (7) versus the LF momentum fraction x. Labels 1 to 4 on the curves refer to the cases $\beta_{bu} = 0.3$ GeV, $m_{ud} = 600$ MeV; $\beta_{bu} = 0.3$ GeV, $m_{ud} = 300$ MeV; $\beta_{bu} = 0.6$ GeV, $m_{ud} = 600$ MeV; $\beta_{bu} = 0.6$ GeV, $m_{ud} = 300$ MeV, respectively.

Figure 2. The lifetime ratios τ_{Λ_b}/τ_B for $\beta = 0.3$ GeV ($r = 0.3$)) (solid line), $\beta = 0.45$ GeV ($r = 1$) (long–dashed line), and $\beta = 0.6$ GeV ($r = 2.37$) (short–dashed line). (a) $m_{ud} = m_u + m_d$, and (b) $m_{ud} \sim \frac{1}{2}(m_u + m_d - m_\pi)$.

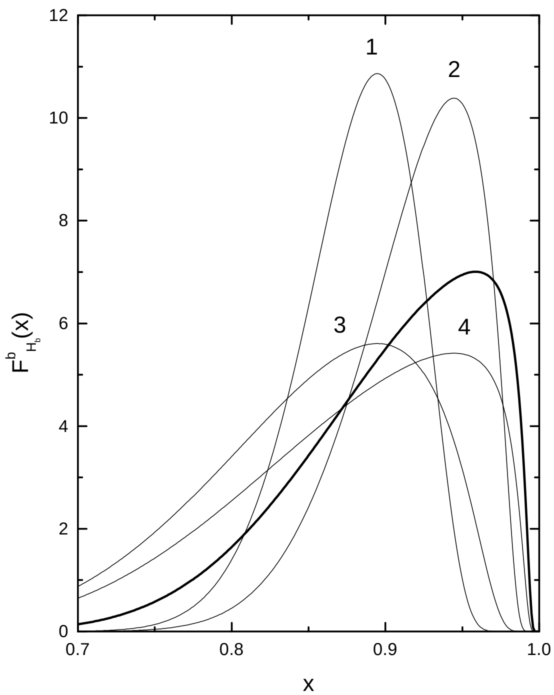

Figure 1

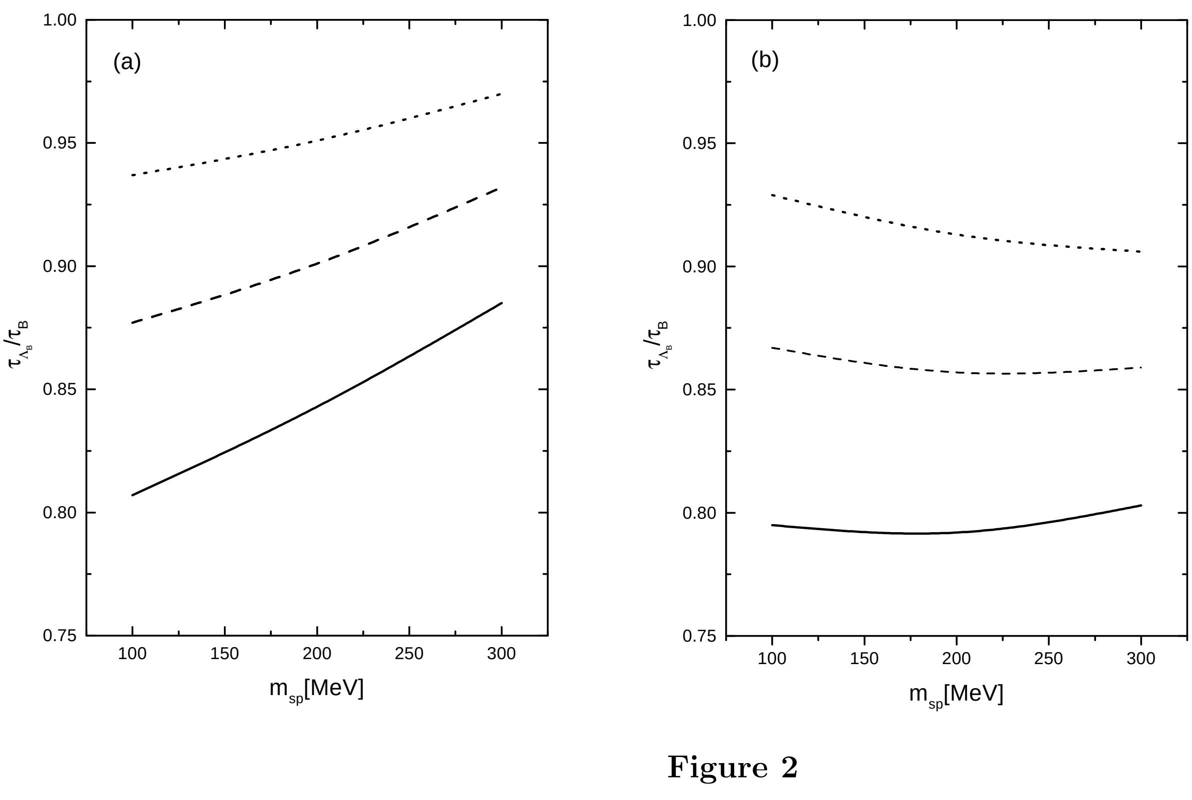

Figure 2